Name ___________________________ Review #3 for Midterm

1. Identify the following aspects of the trig function below and then graph the function.

$$f(x) = -3\sin 2\left(x - \frac{\pi}{2}\right) + 1, \qquad 0 \le x \le 2\pi$$

a. Amplitude: 3

b. Period: $\frac{2\pi}{2} = \pi$

c. Phase shift (horizontal): right $\frac{\pi}{2}$

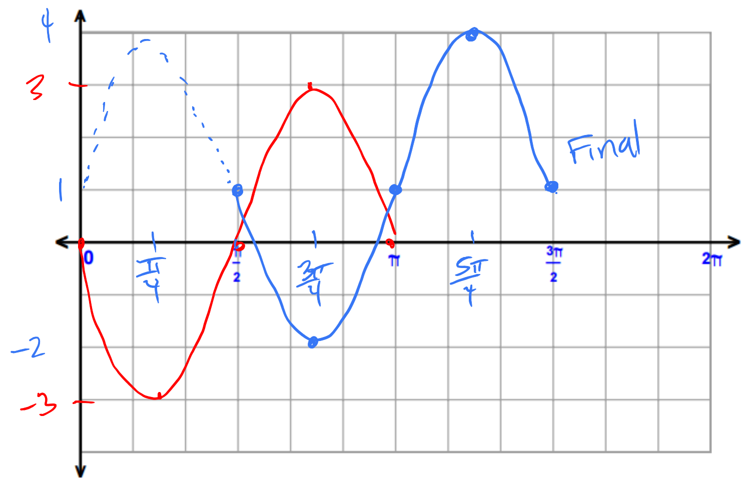
d. Vertical shift: Up 1

e. Express the graphed function as a cosine function:

$f(x) = 3\cos\left(2\left(x - \frac{\pi}{4}\right)\right) + 1$

or

$f(x) = -3\cos\left(2\left(x - \frac{3\pi}{4}\right)\right) + 1$



2. a) Simplify: $\dfrac{1}{\cos t} - \sin t \cdot \tan t$

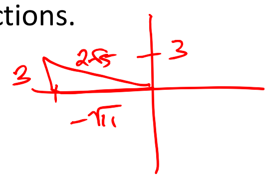
$= \dfrac{1}{\cos t} - \sin t\left(\dfrac{\sin t}{\cos t}\right)$

$= \dfrac{1-\sin^2 t}{\cos t} = \dfrac{\cos^2 t}{\cos t} = \boxed{\cos t}$

b) Prove: $\dfrac{\sin^2\theta}{1-\cos\theta} = 1 + \cos\theta$

$= \dfrac{1-\cos^2\theta}{1-\cos\theta} = \dfrac{(\cancel{1-\cos\theta})(1+\cos\theta)}{\cancel{1-\cos\theta}} = \boxed{1+\cos\theta}$

3. The terminal side of an angle of θ radians passes through the point $\left(-\sqrt{11}, 3\right)$. Find the value of all 6 trig functions.



$3^2 + (-\sqrt{11})^2 = c^2$

$9 + 11 = c^2$

$c = \sqrt{20} = 2\sqrt{5}$

$\sin\theta = \frac{3}{2\sqrt{5}}$ $\quad \csc\theta = \frac{2\sqrt{5}}{3}$

$\cos\theta = \frac{-\sqrt{11}}{2\sqrt{5}}$ $\quad \sec\theta = \frac{2\sqrt{5}}{-\sqrt{11}}$

$\tan\theta = -\frac{3}{\sqrt{11}}$ $\quad \cot\theta = -\frac{\sqrt{11}}{3}$

4. Solve for all real values of $0 \le x \le 2\pi$ in radian measure: $2\cos^2 x - \sin x = 1$

$2(1-\sin^2 x) - \sin x = 1$

Let $u = \sin x$.

$2(1-u^2) - u = 1$

$2 - 2u^2 - u = 1$

$0 = 2u^2 + u - 1$

$0 = (2u-1)(u+1)$

$u = \frac{1}{2}$ $\quad u = -1$

$\sin x = \frac{1}{2}$ $\quad \sin x = -1$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$ $\quad \boxed{x = \frac{3\pi}{2}}$

5. In ΔABC, a = 85, b = 110, and c = 190.

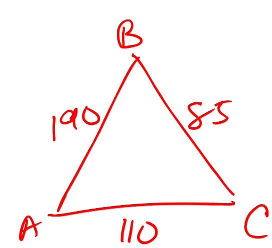
a) Find the measure of angle B.

$110^2 = 190^2 + 85^2 - 2(190)(85)\cos B$

$\cos B = \frac{-31225}{-32300}$

$\boxed{B \approx 15^\circ}$

b) Use the measure of angle B to find the area of ΔABC.



Area $= \frac{1}{2}(190)(85)\sin 15^\circ \approx \boxed{2090}$

Name ___________________________

Review #3 for Midterm

6. Write down a simpler expression that $\sin(\pi - x)$ is equivalent to:

$\sin\pi \cos x - \cos\pi \sin x$

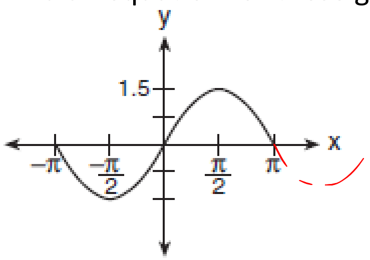
$0 \cdot \cos x - (-1)\sin x = \boxed{\sin x}$

7. Solve for all values of θ in the interval $0 \le \theta < 2\pi$: $\sqrt{3}\cot\theta + 3 = 0$

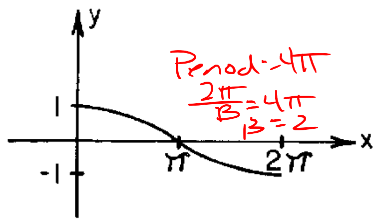
$\cot\theta = -\frac{3}{\sqrt{3}}$ Q2: $\boxed{\theta = \frac{5\pi}{6}}$

$\tan\theta = -\frac{\sqrt{3}}{3}$ Q4: $\boxed{\theta = \frac{11\pi}{6}}$

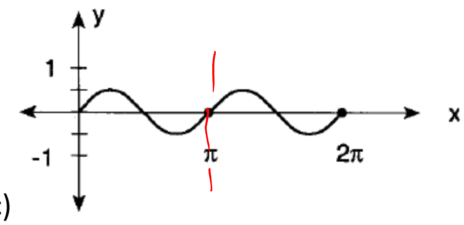
8. Find an equation for these graphs:

a) 

$\boxed{y = 1.5\sin x}$

b) 

$y = \cos(2x)$

c) 

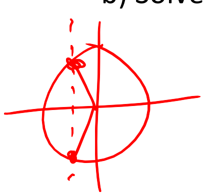
$y = \sin(2x)$

9. Let $\sec x = -2$.

a) What is $\cos x$?

$\cos x = -\frac{1}{2}$

b) Solve the equation above for $x \in [0,2\pi)$



Q2: $\boxed{x = \frac{2\pi}{3}}$

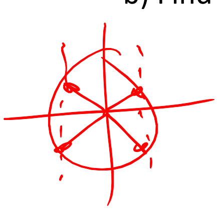
Q3: $\boxed{x = \frac{4\pi}{3}}$

10. Consider the equation $\cos^2 x = \frac{3}{4}$. $\rightarrow \cos x = \pm\frac{\sqrt{3}}{2}$

a) How many solutions do you expect this equation to have, for $x \in [0,2\pi)$? Why?

4. The x-coordinate on the unit circle is $\frac{\sqrt{3}}{2}$ is 2 places and $-\frac{\sqrt{3}}{2}$ in 2 more.

b) Find those solutions!



$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

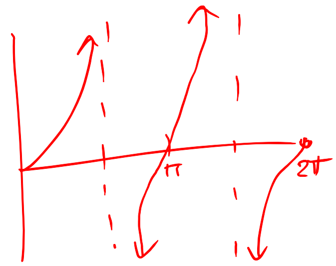
Name ________________________

Review #3 for Midterm

11. Consider the function $f(x) = \tan x$ on the interval $x \in [0, 2\pi]$.

a) What is the period of *f*?

π



b) What are the zeros of *f*?

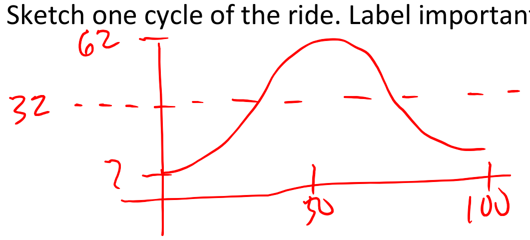
$0, \pi, 2\pi$

c) Where does *f* have vertical asymptotes?

$\frac{\pi}{2}, \frac{3\pi}{2}$

12. Sognefjord is going on a Ferris wheel. Its diameter is 60 feet, and it takes 100 seconds to complete one full counterclockwise rotation. Sognefjord enters the wheel at its lowest point, which is 2 feet off of the ground, when $t = 0$.

a) Sketch one cycle of the ride. Label important points on the x and y axes.



$100 = \frac{2\pi}{B}$

$B = \pi/50$

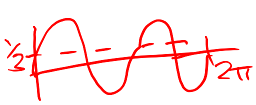
b) Write an equation of the form $f(t) = A\cos(Bt) + C$ to model Sognefjord's height above the ground *t* seconds after she started the ride. Then check in your calculator to make sure your equation matches your graph from a!

$f(t) = -30\cos\left(\frac{\pi}{50}t\right) + 32$

13. How many solutions do you expect the following equations to have, for $x \in [0, 2\pi)$? Why? No need to solve.

a) $\sin(x) = \frac{1}{3}$ 

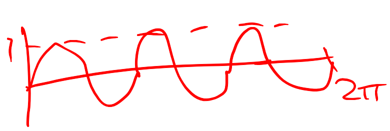
2 sol.

b) $\sin(2x) = \frac{1}{3}$ 

4 sol.

c) $\sin(x) = 3$ 

0 sol.

d) $\sin(3x) = 1$ 

3 sol.

Name ___________________________

Review #3 for Midterm

14. Let $\cos x = \frac{1}{3}$ where x terminates in quadrant 4.

a) Find the exact value of $\sin x$. Be sure to draw a picture to make sure your answer makes sense!

$\cos^2 x + \sin^2 x = 1$

$\left(\frac{1}{3}\right)^2 + \sin^2 x = 1 \to \frac{1}{9} + \sin^2 x = 1$

$\sin^2 x = 8/9 \to \sin x = \pm\sqrt{\frac{8}{9}}$

$\sin x = \boxed{-\frac{2\sqrt{2}}{3}}$

neg. b/c Q4

b) Find the exact value of $\sin(2x)$.

$= 2\sin x\cos x = 2\left(\frac{-2\sqrt{2}}{3}\right)\left(\frac{1}{3}\right) = \boxed{\frac{-4\sqrt{2}}{9}}$

15. Find the exact value of the following:

a) $\sin\left(\frac{2\pi}{3}\right)$ $= \frac{\sqrt{3}}{2}$

b) $\cos\left(\frac{11\pi}{6}\right)$ $= \frac{\sqrt{3}}{2}$

c) $\sin\left(\frac{7\pi}{4}\right)$ $= -\frac{1}{\sqrt{2}}$

d) $\cos\left(\frac{3\pi}{2}\right)$ $= 0$

e) $\tan\left(\frac{5\pi}{6}\right)$ $= -\frac{1}{\sqrt{3}}$

f) $\cot\left(\frac{\pi}{2}\right)$ $= \frac{0}{1} = 0$

g) $\sec\left(\frac{\pi}{6}\right)$ $= \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$

h) $\csc\left(\frac{\pi}{4}\right)$ $= \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

16. Prove that:

a) $\frac{\sec x}{\cot x + \tan x} = \sin x$

$\frac{\frac{1}{\cos x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}} = \frac{\frac{1}{\cos x}}{\frac{\cos^2 x + \sin^2 x}{\sin x \cos x}} = \frac{1}{\cancel{\cos x}} \cdot \frac{\sin x \cancel{\cos x}}{\underbrace{\cos^2 x + \sin^2 x}_{=1}}$

$\boxed{= \sin x}$

b) $\frac{1+\cot x}{\tan x + 1} = \cot x$

$\frac{1 + \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} + 1} = \frac{\frac{\sin x + \cos x}{\sin x}}{\frac{\sin x + \cos x}{\cos x}} = \frac{\cancel{\sin x + \cos x}}{\sin x} \cdot \frac{\cos x}{\cancel{\sin x + \cos x}}$

$= \frac{\cos x}{\sin x} = \boxed{\cot x}$